A Method of Compliance Control of Redundant Manipulators

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A compliance control method of redundant manipulators is presented. This method is based on the new stiffness model, which allows to modulate accurate joint stiffness of realizing the end effector stiffness to be varied with task requirements. A control model is developed and implemented by the proposed method in a three degree of freedom planar redundant manipulator. Also its effectiveness is validated.

Key Words: Compliance Control, Stiffness Model, Orthogonal Stiffness Decomposition Control, Redundant Manipulator, Induced Stiffness Obtained from Configuration Change and Force

1. Introduction

In robot manipulators interacting with the environment, the capability of compliance control is prerequisite. Up to now, though a lot of approaches have been proposed, the compliance control can be classified into two main categories : the hybrid position/force control (Raibert, et. al., 1981) and the impedance control (Hogan, 1985). In this paper, we address the stiffness control regarded as one of the impedance control schemes, especially that of redundant manipulators.

The stiffness control originates from Salisbury (1980). In the case of redundant manipulators, this controller has a limitation in that the null space motion is left uncontrolled. The end effector can, however, be stationary in unconstrained situation if joint friction is large enough to keep the null space motion from drifting and dynamic disturbances are assumed to be negligible. On the contary, if the stiffness controlled manipulator

contacts with the environment exerting the static force at the end effector, the manipulator cannot preserve its configuration any more and will collapse. The static force generates some disturbance torque, which in turn makes the joint configuration unstable.

There have been a lot of investigations concerning the stiffness control of redundant manipulators (Kaneko, et. al., 1988; Kim. et. al., 1992; Yokoi, et. al., 1992), and they provide their own methods of specifying the joint stiffness corresponding to the given taskspace stiffness. However, these methods are basically derived from the stiffness model of Salisbury (1980), that is, the congruence mapping of stiffnesses (Strang, 1988). In this model, there is no consideration on the effect of the static force. According to Yi et. al., (1991) the stiffness characteristics at the joint depend on the direction of end effector force and manipulator configuration, and Mussa-Ivaldi and Hogan(1991) derived a stiffness model considering the effect of static force although they did not deal with the stiffness specification at the taskspace. Their investigations suggest that the previous model might not be sufficient to describe the stiffness relation, and the effect of static force be required to be included in the model.

In this paper, we present a new stiffness model derived from static force equilibrium. In this

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model, the effect of static force is introduced in terms of equivalent stiffness, called induced stiffness obtained from configuration change and force (ISOC), and the accurate relationship between force and deflection is represented. The model suggests what torque should be imposed to realize the desired stiffness and restoring force at the end effector. Conversely, the model makes it possible to evaluate the resultant joint stiffness. Based on the developed stiffness model, we propose a stiffness control scheme, called orthogonal stiffness decomposition control (OSDC). OSDC is implemented in a three-degree of freedom planar redundant manipulator adopting the tendon driven method and its effectiveness is confirmed by conducting several experiments.

2. Stiffness Modeling

In this section, we will establish the stiffness model describing the relations among taskspace stiffness, joint stiffness and static force. *Kinematic redundancy* is usually used to indicate the excess of actively controlled dof at the joint space with respect to that of the taskspace. Here, the taskspace kinematics is described by an *N*-dimensional position vector $x = [x_1 \cdots x_N]^T$ that specifies the location of the end effector with respect to an absolute coordinate frame. The manipulator configuration is fully described by an *n*-dimensional vector of generalized coordinates $q = [q_1 \cdots q_n]^T$. Redundancy is expressed by the inequality $N \le n$. The forward kinematics in a manipulator can be represented with a vector map

$$x = F(q) \tag{1}$$

from configuration to end effector position, where $F \in \Re^N$ is a vector function assumed to be twice differentiable in the entire workspace. The differential transformation from joint displacement to end effector displacement is

$$dx = J_q(q) \, dq \tag{2}$$

where $J_q(q) = \partial F(q) / \partial q \in \mathbb{R}^{N \times n}$ is the Jacobian of the manipulator. Assuming the frictional and dynamic forces are compensated for or small enough to be neglected, we can compute the joint torque $\tau = [\tau_1 \cdots \tau_n]^T$ necessary to apply an end effector force $f = [f_1 \cdots f_N]^T$ according to the static relation

$$\tau = J_q^T(q) f \tag{3}$$

Also, it has been found useful to be able to superimpose bias force on the stiffness behavior (Salisbury, 1980) and thus, we suppose that the manipulator be at static equilibrium with its bias force f_0 , where we have $x_0 = F(q_0)$ and $\tau_0 = J_q^T$ $(q_0) f_0$. Now, if the manipulator is deflected by dxand going to increase the exerting force by df as illustrated in Fig. 1, the manipulator should move to another static equilibrium at the changed configuration $q_0 + dq$ with the changed force f and torque τ such as

$$\tau = J_q^T (q_0 + dq) f \tag{4}$$

Also, the definition of stiffness provides us with the following auxiliary stiffness relations.

$$\tau = \tau_0 + K_q \left(q_0 - q \right) \tag{5}$$

$$f = f_0 + K_x(x_0 - x)$$
 (6)

where $K_q \in \mathbb{R}^{n \times n}$ and $K_x \in \mathbb{R}^{N \times N}$ are the stiffnesses at joint and taskspace, respectively. Differentiating Eqs. (5) and (6), we get

$$d\tau = -K_q dq \tag{7}$$

$$df = -K_x dx \tag{8}$$

The Jacobian matrix at the righthand side of Eq. (4) can be expanded using Taylor series such as



Fig. 1 Model of redundant manipulator

$$J_q^T(q_0 + dq) \approx J_q^T(q_0) + \frac{\partial f_q^T(q_0)}{\partial q} dq \qquad (9)$$

Substituting Eq. (9) into the righthand side of Eq. (4) yields the result

$$J_q^T(q_0 + dq) f$$

= $J_q^T(q_0) f + \frac{\partial T_q^T(q_0)}{\partial q} dq f$ (10)

In addition, the last term in the righthand side of Eq. (10) is rearranged as

$$\frac{\partial J_q^T(q_0)}{\partial q} dq f = \varDelta(q_0, f) dq \tag{11}$$

where the *i*th row and the *j*th column of $\Delta(q, f)$ is obtained as follows:

$$\Delta(q, f)_{(i,j)} = \sum_{k=1}^{N} \frac{\partial^2 F_k}{\partial q_i \partial q_j} f_k$$
(12)

Using Eqs. (7), (8) and (11), Eq. (4) will be

$$-K_q dq = J_q^T(q_0) df + \varDelta(q_0, f) dq \qquad (13)$$

and the chain rule gives us

$$df = -K_x J_q(q_0) \, dq \tag{14}$$

Substituting Eq. (14) into Eq. (13), we get the result

$$K_q = J_q^{T}(q_0) K_x J_q(q_0) - \varDelta(q_0, f)$$
(15)

The above Eq. (15) describes how to specify the joint stiffness to realize the desired taskspace stiffness K_x . According to the conventional approaches (Salisbury, 1980) there is no consideration of $\Delta(q_0, f)$ and the joint stiffness is just determined by the congruence transformation of obtained from configuration change and force (ISOC) in this paper, and it is a function of the exerting force and the configuration of the manipulator. Here, note that the exerting force f is the summation of bias force f_0 and incremental force df caused by the takspace stiffness. If $f_0 = O$ an⁴ df is negligible, Eq. (15) comes to the conventional stiffness model (Salisbury, 1980). However, as long as the manipulator is desired to exert the controlled force in the reasonable range of motion, ISOC may not be ignored.

Conversely, let us derive the resultant tasksace stiffness for the given joint stiffness K_q and end effector force f. Eq. (13) can be reformulated as follows:

$$- [K_q + \varDelta(q_0, f)] dq = J_q^T(q_0) df$$
 (16)

Assuming that Det $[K_q + \Delta(q_0, f)] \neq 0$ and inverting the premultiplying term of dq yields

$$dx = -J_q(q_0) [K_q + \varDelta(q_0, f)]^{-1} J_q^T(q_0) df$$
$$\triangleq -K_x^{-1} df \tag{17}$$

Therefore, the taskspace stiffness K_x is written

$$K_{x} = \{J_{q}(q_{0}) [K_{q} + \varDelta(q_{0}, f)]^{-1} J_{q}^{T}(q_{0})\}^{-1}$$
(18)

Finally, assuming G(q) is the joint torque equivalent to the gravity force acting on the links of the manipulator, Eqs. (15) and (18) will be changed to

$$K_{q} = J_{q}^{T}(q_{0}) K_{x} J_{q}(q_{0}) - \varDelta(q_{0}, f) - \varDelta_{g}(q_{0}, f_{g})$$
(19)

$$K_{x} = \{J_{q}(q_{0}) [K_{q} + \mathcal{\Delta}(q_{0}, f) + \mathcal{\Delta}_{g}(q_{0}, f_{g})]^{-1} J_{q}^{T}(q_{0})\}^{-1}$$
(20)

where f_g denotes the gravity force, and $\Delta_g(\cdot, \cdot)$ is ISOC induced by the gravity force such as

$$\Delta_g(q_0, f_g) = \frac{\partial G(q_0)}{\partial q}$$
(21)

Now, we let

$$K_{qe} = K_q + \varDelta(q_0, f) + \varDelta_g(q_0, f_g)$$
(22)

and examine Eqs. (19) and (20). First, it is noticed from Eq. (20) that the effective stiffness at the joint comes to be K_{qe} . K_{qe} is the resultant stiffness at the joint and determines the actual responsive torque for the joint displacement. Remembering that the stability of an elastic system is determined by the characteristics of the stiffness matrix (mathematically, positive definiteness), it can be stated that the stability of the joint configuration depends on K_{qe} . Although the stable joint servo stiffness K_q is given, therefore, the joint configuration may not be stable if the effective joint stiffness K_{qe} comes to be negative definite because of ISOC. Second, if we servo the joint servo stiffness according to Eq. (19), ISOC is precompensated at the joint and the effective stiffness at the joint will be computed as follows :

$$K_{x} = \{J_{q}(q_{0}) K_{q}^{-1} J_{q}^{T}(q_{0})\}^{-1}$$
(23)

This is the same as the stiffness model proposed by the previous works, where ISOC is not considered (Salisbury, 1980; Kaneko, et. al., 1988; Kim, et. al., 1992; Yokoi, et. al., 1992). Thus, the stiffness control problem can be treated in the same way as the previous works, which is going to be discussed in the next section.

3. Orthogonal Stiffness Decomposition Control

In the previous section, we presented a new stiffness model to describe the relations between the exerting force and the corresponding stiffness ses. Equation (20) offers a *forward stiffness computation model* and Eq. (19) *a backward stiffness computation model*, that is, a *stiffness control model*. Based upon these formulation, a stiffness control method named *orthogonal stiffness decomposition control* (OSDC) is proposed, which provides a proper way to compensate ISOC and specify null stiffness.

Let us suppose that the manipulator at an initial configuration $q = q_0$ is deflected and exerts the end effector force f. If the joint stiffness is servoved according to Eq. (19) such as

$$K_{q} = J_{q}^{T}(q_{0}) K_{x} J_{q}(q_{0}) - \varDelta(q_{0}, f) - \varDelta_{g}(q_{0}, f_{g})$$
(24)

then, according to Eq. (22), the effective servo stiffness at the joint will be

$$K_{qe} = J_q^T(q_0) K_x J_q(q_0)$$
(25)

and it can be perceived that the stiffness model comes to that of the previous works (Salisbury, 1980; Kaneko, et. al., 1988; Kim, et. al., 1992; Yokoi, et. al., 1992). Now, the behavior of the manipulator depends upon the characteristic of K_{qe} , but it is known to be singular in the case of a redundant manipulator and the manipulator servoed according to Eq. (24) cannot be stable. Therefore, the problem comes to the derivation of a positive definite joint servo stiffness that brings about the specified taskspace stiffness. In this paper, we employ the following method (Kim, et. al., 1992).

We let $\tilde{K}_q = J_q^T(q_0) K_x J_q^T(q_0)$ and consider the method to include the stiffness term that makes K_{qe} positive definite and that is perpendicular to \tilde{K}_q .

If \tilde{K}_q is decomposed by using the similarity

transform (Strang, 1988), a diagonal matrix A_p having eigenvalues as its diagonal elements is obtained.

$$\Lambda_p = H^{\mathsf{T}} \tilde{K}_q H \tag{26}$$

$$= \operatorname{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n)$$
(27)

where the columns of the orthogonal matrix H $(H^{-1}=H^T)$ are the eigenvectors corresponding to the eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$ of \tilde{K}_q , respectively, and

$$\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_N \ge \lambda_{N+1} = \cdots = \lambda_n = 0 \qquad (28)$$

Here, a new matrix A_h is defined with the non -zero diagonal elements $\tilde{\lambda}_i \ge 0$ $(N+1 \le i \le n)$ instead of zero diagonals for Eq. (27).

$$\Lambda_{h} = \operatorname{diag}(0, 0, \cdots, \tilde{\lambda}_{N+1}, \cdots, \tilde{\lambda}_{n}) \qquad (29)$$

Even if the zero eigenvalues λ_i $(N+1 \le i \le n)$ is shifted to arbitrary positive values $\tilde{\lambda}_i$ the orthogonal matrix H remains the same, since the matrix A_h is always orthogonal to the matrix A_p . Thus, a new diagonal matrix A is defined as the sum of A_p and A_h .

$$A = A_p + A_h \tag{30}$$

If Λ is transformed reversely, the result consequently comes to be

$$HAH^{T} = J_{q}^{T}K_{x}J_{q} + K_{n}$$

$$\tag{31}$$

where $K_n \cong H \Lambda_h H^T$. Then,

$$(K_n)^T (J_q^T K_x J_q) = O$$
(32)

The matrix K_n , which is termed as *null stiff*ness, enables us to directly assign new nonzero eigenvalues $\tilde{\lambda}_i$ $(N+1 \le i \le n)$ in the direction perpendicular to the hyperplane in which \tilde{K}_q lies. By adding K_n , the effective joint stiffness K_{qe} becomes positive definite but K_n does not change the resultant taskspace stiffness as it is orthogonal to \tilde{K}_q . The $\tilde{\lambda}_i$'s determine the magnitude of the stiffness controlling the null motion in the direction of redundancy, which corresponds to the remaining eigenvectors of the matrix H.

Finally, we can obtain the complete joint stiffness from OSDC as follows:

$$K_q = J_q^T K_x J_q + K_n - \mathcal{\Delta}(q_0, f) - \mathcal{\Delta}_g(q_0, f_g)$$
(33)

While the intended operation of this controller is in contact with the environment there are situations where we operate out of contact with it. It is therefore necessary to add damping effect, in the same way as the taskspace stiffness is specified. We introduce the individual joint damping $K_{qd} \in \Re^{n \times n}$ such as

$$K_{qd} = \alpha \ H \ \text{diag}(\lambda_1, \lambda_2, \cdots, \tilde{\lambda}_{N+1}, \cdots, \tilde{\lambda}_n) \ H^T (34)$$

where α is a scalar scale coefficient that determines the strength of damping. Therefore, the final form of the applied torque is given by the expression

$$\tau = K_q (q_0 - q) + K_{qd} (\dot{q}_0 - \dot{q}) + J_q^T f_0$$
(35)

where \dot{q}_{d} and \dot{q} are the desired and current joint velocity, respectively.

4. Experimental Verification

To examine the effectiveness of the proposed approach, several experiments were performed using a planar three-dof redundant manipulator.

4.1 Outline of experimental setup

The manipulator was configured as a planar three-link structure and to move on the plane orthogonal to the gravity field as illustrated in Figs. 2 and 3, where the values of l_1 , l_2 and l_3 are set 106, 65, 65mm, respectively.

The three joints were remotely actuated via tendons. Six torque controlled DC servo motors (rated torque 1.4Nm, torque constant 2.1Nm/A, and rated speed 60rpm including harmonic reduction gear of 50 : 1), were used to provide commanded tensions for the tendons. The overall system architecture is illustrated in Fig. 4.

The hardware was composed of two 68030 based single board computers (Motorola MC68030 CPU) with floating point unit in a VMEbus card cage. A SUN/SPARC workstation was used as a host computer. It had a D/A converter with 16 channels and an A/D converter with differential 16 channels to interface with sensors. Joint positions were measured by potentiometers directly attached at the joints, and joint torques were computed by measured ten-



Fig. 2 Three-dof redundant manipulator



Fig. 3 Schematic of three-dof redundant manipulator



Fig. 4 Overall system architecture

sions and the tension-torque relationship of the coupled tendon driven system (Hirose, et. al., 1991). The controllers were implemented discretely, and control programs were written in C language with a few 68030 assembly codes. The programs were developed on a workstation in the UNIX environment using a commercial realtime software development tool called VRTX velocity (Ready Systems Inc., 1991). The actuator level control and stiffness control loop ran at 400Hz and 100Hz, respectively.

4.2 Experiments

In the first experiment, the effect of ISOC was verified. We let the tip of the manipulator brought contact with a single axis force sensor (Bongshin load cell, 20kgf) and a 2N bias force was given. In this situation, the depth of contact was increased about 0.06m very slowly along the contact surface of the force sensor, expecting to exert the static contact force corresponding to the displacement offset similar to Fig. 1. The taskspace stiffness was given by

$$K_x = \text{diag}(100, 100) (\text{N/m})$$
 (36)

To show the importance of ISOC proposed in our stiffness model, two control methods were applied: the one was OSDC and the other was the controller which did not compensate ISOC. For convenience, the latter was called the conventional controller because it started from the stiffness model of the previous works. In the experiments, the eigenvalue of null stiffness was set to

$$\lambda_3 = 0.045$$
 (37)

Thus, the joint stiffness by OSDC became

$$K_q = J_q^T K_x J_q + H \text{ diag}(0, 0, 0.045) H^T - \Delta(q_0, f)$$
(38)

and that of the conventional controller was

$$K_q = J_q^T K_x J_q + H \operatorname{diag}(0, 0, 0.045) H^T$$
(39)

Also, α was set to 0.2.

Beforehand, we computed the minimum eigenvalues of the effective joint stiffness using Eq. (22) for the two control methods. As shown in Fig. 5, the minimum eigenvalue of the conventional controller came to be less than zero. It means that though the stable joint stiffness was specified in the conventional control method, the effective joint stiffness became negative definite due to ISOC and the configuration of the manipu-



Fig. 5 Minimum eigenvalues of the effective joint stiffness



Fig. 6 Desired configuration of manipulator



Fig. 7 Configuration of manipulator by conventional controller

lator was expected to be unstable.

The instability phenomena could be observed in the results of the experiments. Figures 6, 7 and 8 show the desired configuration of the manipulator, the configuration of the manipulator of the conventional controller, and that of the OSDC during the experiments, respectively. Here, the desired configuration represents that of minimum



Configuration of manipulator by proposed con-Fig. 8 troller

potential energy. To determine the desired configuration of the manipulator, we introduced the following performance function

$$H(q) = \frac{1}{2} (q - q_0)^{T} K_q (q - q_0)$$
(40)

Using the technique proposed in (Chang, 1987), we computed the desired joint configuration to satisfy the constraints of the end effector position and minimum potential energy. As illustrated in these figures, the conventional controller could not preserve the configuration of the manipulator and its configuration was far from the desired one. With the proposed method, on the contrary, the manipulator showed stable behavior and also was close to the desired configuration during the experiments with the proposed method. The effect of ISOC could be apparently perceived in the data of contact forces. The desired contact force is going to be the sum of the bias force and the force generated by stiffness such as

$$f_d = K_x (x - x_0) + f_0 \tag{41}$$

where f_d is the desired contact force. As given in Fig. 9, the experimental results employing the conventional method could not track the desired force trajectory. As showin in Fig. 10, the proposed controller, on the other hand, exerted the desired static force successfully.

In the second experiment, the compliance behavior of the manipulator was tested. The manipulator was driven along the planned trajectory with a given taskspace stiffness. Along the traveling path, the manipulator was forced to contact with the environment intentionally and



Fig. 9 Contact force of conventional controller



Fig. 10 Contact force of proposed controller



Fig. 11 Schematic of experiment

the contact force was measured. The force sensor used in the first experiment, was placed in the taskspace parallel to the X-Y plane as shown in Fig. 11. The nominal trajectory was given as a circular trajectory with 0.01m radius and 0.1Hz period, which is low enough to neglect the inertial



Fig. 12 Contact motion trajectory of conventional control method



Fig. 13 Contact motion trajectory of proposed control method

effect. Similar to the first experiment, two control methods were applied : the controller compensated for ISOC and the one without compensation. The taskspace stiffness was given by

$$K_x = \text{diag}(400, 400) (\text{N/m})$$
 (42)

The eigenvalue of null stiffness was set to be $\lambda_3 = 0.045$, and the strength of damping, α , was 0.2. The traveling paths of the end effector during the experiments are illustrated in Figs. 12 and 13. It could be observed that as the deflection gets larger the manipulator controlled by the conventional method had difficulties keeping track of the nominal trajectory. We noticed the similar fact in



Fig. 14 Measured contact force of conventional control method



Fig. 15 Measured contact force of proposed control method

the plots of measured contact forces given in Figs. 14 and 15. The conventional control method showed poor tracking performance of force, and vice versa.

5. Conclusions

A compliance control method of redundant manipulators has been proposed. Through stiffness modeling, we have formulated the equations to describe the relations between force and stiffness, where a new term called *induced stiffness obtained from configuration change and force* (ISOC) was introduced. ISOC can be considered as the resultant effect of the static force expressed in terms of stiffness. The proposed control method based on the developed model, provides a feasible way of controlling stiffness of redundant manipulators. The importance of ISOC and the effectiveness of the proposed control method have been confirmed by experiments.

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